CHAPTER 5

OBLIQUE TRIANGLES

LEARNING OBJECTIVES

Upon completion of this chapter, you should be able to do the following:

1. Apply the Law of Sines to solve oblique triangles given one side and two angles or two sides and an angle opposite one of them.

2. Apply the Law of Cosines to solve oblique triangles given two sides and the included angle or all three sides.

3. Find the area of an oblique triangle.

INTRODUCTION

The two previous chapters primarily dealt with properties of right triangles in solving trigonometric measurements and functions. In this chapter we will apply properties of oblique triangles in solving trigonometric measurements and functions. *Oblique triangles* are triangles containing no right angles. Oblique triangles are made up of either three acute angles or two acute angles and one obtuse angle. *Acute angles* have measures between 0° and 90°. *Obtuse angles* have measures between 90° and 180°.

In *Mathematics,* Volume 1, a method for solving problems involving oblique triangles was introduced. The method employed the procedures of dividing the original triangle into two or more right triangles and using the properties of right triangles in problem solving.

This chapter develops two methods or laws dealing directly with oblique triangles. The methods consider the parts of the triangle that are given. The four standard cases for solving oblique triangles are as follows:

Case 1. One side and two angles

Case 2. Two sides and an angle opposite one of them

Case 3. Two sides and the included angle

Case 4. All three sides

Also included in this chapter are problems concerning the area of a triangle, which combine the area formula of plane geometry with trigonometric properties.

METHODS OF SOLVING OBLIQUE TRIANGLES

This section is concerned with the development and proofs of the Law of Sines and the Law of Cosines. The four standard cases for solving oblique triangles use applications of these laws.

LAW OF SINES

Law of Sines. *The lengths of the sides of any triangle are proportional to the sines of their opposite angles*; that is,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

PROOF: Refer to the oblique triangle shown in figure 5-1, view A. Let h be the length of the perpendicular from angle A to the side opposite angle A. Considering the two right triangles formed by h, we obtain

$$\sin B = \frac{h}{c} \text{ or } h = c \sin B$$

and

$$\sin C = \frac{h}{b} \text{ or } h = b \sin C$$

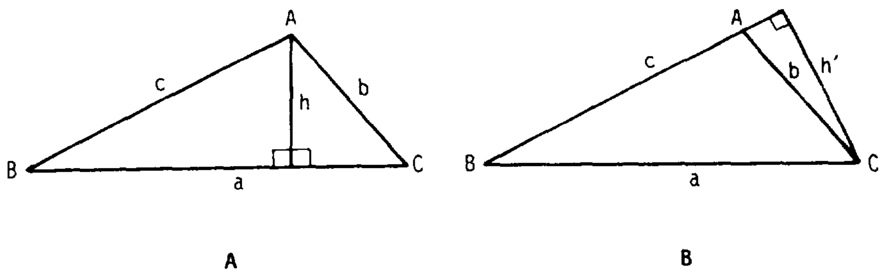

Figure 5-1.—Development of Law of Sines.

Equating these two values of h, we have

$$c \sin B = b \sin C$$

or in an equivalent form, we have

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

Now, if we redraw the oblique triangle in figure 5-1, view A, by extending the length of side c until it forms a right angle (is perpendicular) with a line, h', from angle C (see fig. 5-1, view B), then from the newly formed triangle, we obtain

$$\sin B = \frac{h'}{a} \text{ or } h' = a \sin B$$

and

$$\sin (180° - A) = \frac{h'}{b} \text{ or } h' = b \sin (180° - A)$$

Since

$$\sin (180° - A) = \sin A$$

then by substituting $\sin A$ for $\sin (180° - A)$ and equating values of h', we have

$$a \sin B = b \sin A$$

or in an equivalent form

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

But

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Therefore,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Case 1. One Side and Two Angles

When one side and two angles of a triangle are given, the third angle can be found since the sum of the angles equals 180°; that is, $A + B + C = 180°$. Then the Law of Sines can be used to find the two remaining sides.

EXAMPLE: Solve the remaining parts of triangle ABC, given $c = 5$, $B = 30°$, and $C = 97° \ 30'$. Give side accuracy to one decimal place.

SOLUTION: Refer to figure 5-2. Since

$$A + B + C = 180°$$

then

$$A + 30° + 97° \ 30' = 180°$$

$$A = 180° - 30° - 97° \ 30'$$

$$= 52° \ 30'$$

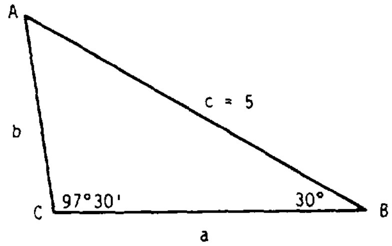

Figure 5-2.—Case 1. One side and two angles.

By the Law of Sines,

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

we obtain

$$\frac{a}{\sin 52° \ 30'} = \frac{5}{\sin 97° \ 30'}$$

$$a = \frac{5 \sin 52° \ 30'}{\sin 97° \ 30'}$$

$$= \frac{5(0.79335)}{0.99144}$$

$$= 4.0$$

We will use the Law of Sines again to solve for the length of side b:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

so,

$$\frac{b}{\sin 30^\circ} = \frac{5}{\sin 97^\circ 30'}$$

$$b = \frac{5 \sin 30^\circ}{\sin 97^\circ 30'}$$

$$= \frac{5(0.50000)}{0.99144}$$

$$= 2.5$$

EXAMPLE: The base of flagpole standing vertically on a hill is inclined at an angle of 15° with the horizontal. A man standing 200 feet downhill from the base of the flagpole notes that his line of sight to the top of the flagpole makes an angle of 40° with the horizontal. How high, to the nearest foot, is the flagpole?

SOLUTION: Refer to figure 5-3. In triangle *ABC* we find

$$A = 40^\circ - 15^\circ$$

$$= 25^\circ$$

From right triangle *ADC* we find

$$C = 180^\circ - 40^\circ - 90^\circ$$

$$= 50^\circ$$

Applying the Law of Sines, we obtain

$$\frac{a}{\sin 25^\circ} = \frac{200}{\sin 50^\circ}$$

$$a = \frac{200 \sin 25^\circ}{\sin 50^\circ}$$

$$= \frac{200(0.42262)}{0.76604}$$

$$= 110 \text{ feet}$$

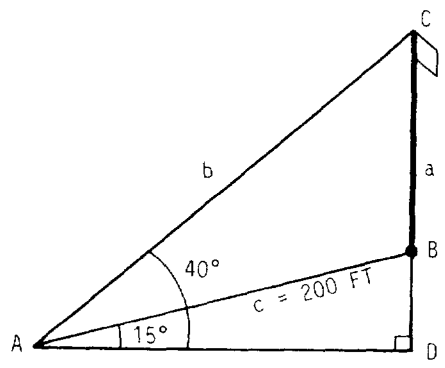

Figure 5-3.—Case 1. Flagpole problem.

Case 2. Two Sides and an Angle Opposite One of Them

Case 2 is sometimes referred to as the *ambiguous case* since two triangles, one triangle, or no triangle

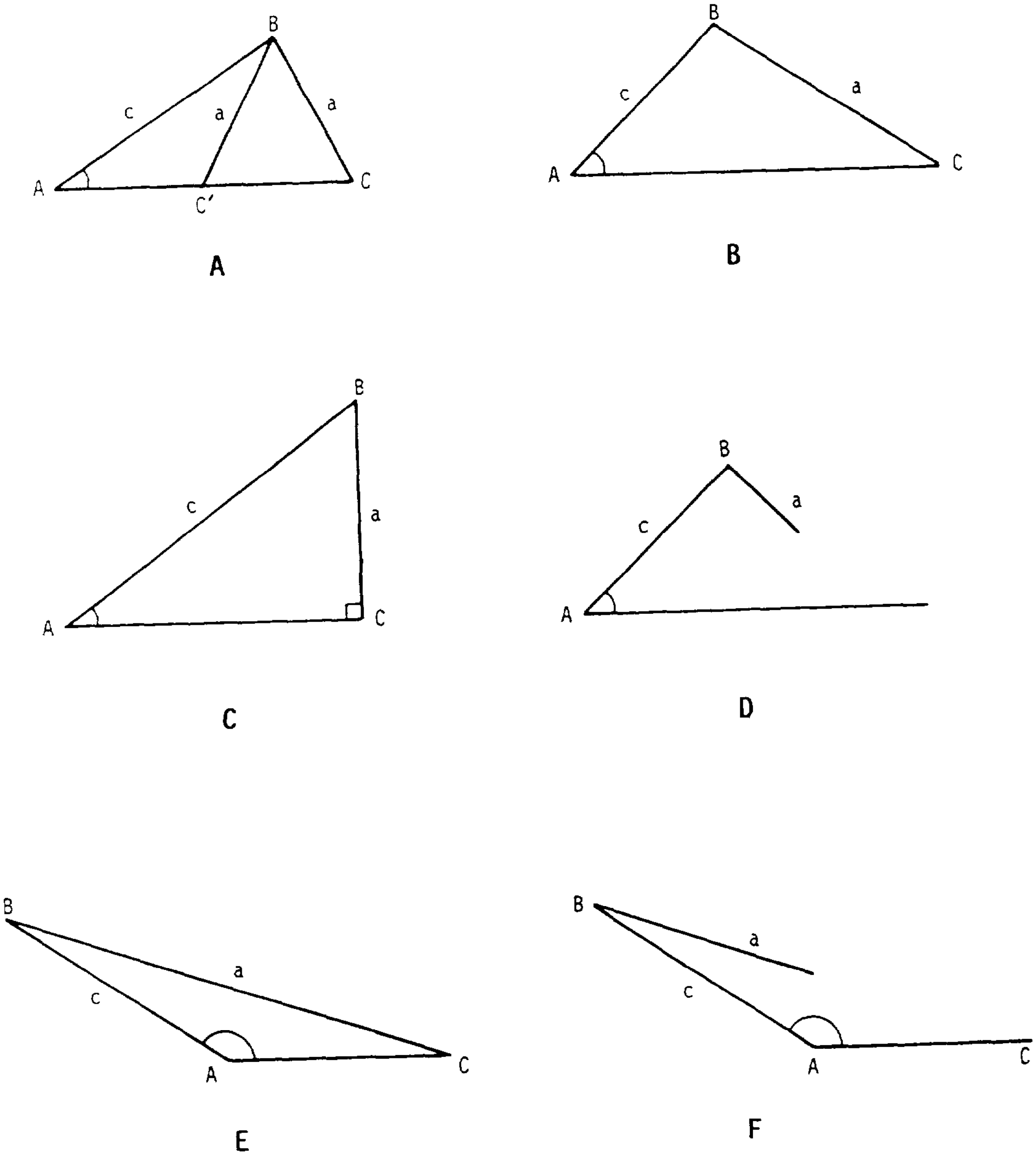

Figure 5-4.—Case 2. Two sides and an angle opposite one of them.

may result from data given in this form. Consider triangle ABC in figure 5-4. Assuming we are given angle A and sides a and c, the following situations may exist:

For acute angle A:

1. If $a < c$ and $\sin C < 1$, then two possible triangles exist; one triangle comprises the acute angle C and the other triangle comprises the obtuse angle $C' = 180° - C$. See figure 5-4, view A.

2. If $a \geq c$, then one triangle exists. See figure 5-4, view *B*.

3. If $a < c$ and $\sin C = 1$, then one right triangle exists. See figure 5-4, view C.

4. If $a < c$ and $\sin C > 1$, then no triangle is determined. See figure 5-4, view D. (This should be obvious since in the previous chapter we learned that the sine of an angle may have values only between 0 and 1.)

For obtuse angle A:

1. If $a > c$, then one triangle exists. See figure 5-4, view E.

2. If $a \leq c$, then no triangle is determined. See figure 5-4, view F.

When two sides and an angle opposite one of them are given, we can solve for the remaining parts of the triangle using the Law of Sines. Sketches can be helpful.

EXAMPLE: Solve the triangle or triangles if they exist, given $B = 45°$, $b = 3$, and $c = 7$.

SOLUTION: Using the Law of Sines,

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

we have

$$\frac{3}{\sin 45°} = \frac{7}{\sin C}$$

$$\sin C = \frac{7 \sin 45°}{3}$$

$$= \frac{7(0.70711)}{3}$$

$$= 1.64992$$

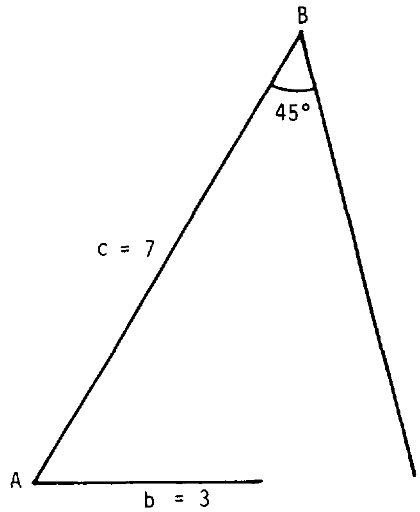

Figure 5-5.—Case 2. Acute angle A with $b < c$ and $\sin C > 1$.

Since the sine of an angle cannot exceed 1, then we conclude that no triangle exists. Refer to figure 5-5. Notice that $b < c$ and $\sin C > 1$.

EXAMPLE: Solve the triangle or triangles if they exist, given $A = 22°$, $a = 5.4$, and $c = 14$. Give angle accuracy to the nearest minute and side accuracy to one decimal place.

SOLUTION: Using the Law of Sines,

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

we obtain

$$\frac{5.4}{\sin 22°} = \frac{14}{\sin C}$$

$$\sin C = \frac{14 \sin 22°}{5.4}$$

$$= \frac{14(0.37461)}{5.4}$$

$$= 0.97121$$

$$C = 76° 13'$$

Since the side opposite the known angle is smaller than the other given side, that is, $a < c$, and $\sin C < 1$, then two possible triangles exist. One triangle is ABC and the other is $AB'C'$. Refer to figure 5-6. Hence,

$$C' = 180° - C$$

$$= 180° - 76° 13'$$

$$= 103° 47'$$

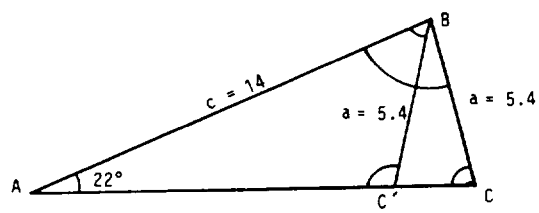

Figure 5-6.—Case 2. Acute angle A with $a < c$ and $\sin C < 1$.

Solving triangle ABC first, we find angle B by

$$B = 180° - (A + C)$$

$$= 180° - (22° + 76° 13')$$

$$= 81° 47'$$

and by the Law of Sines,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

we find the length of side b to be

$$\frac{5.4}{\sin 22°} = \frac{b}{\sin 81° 47'}$$

$$b = \frac{5.4 \sin 81° 47'}{\sin 22°}$$

$$= \frac{5.4(0.98973)}{0.37461}$$

$$= 14.3$$

Now solving triangle $AB'C'$, we find angle B' by

$$B' = 180° - (A + C')$$

$$= 180° - (22° + 103° 47')$$

$$= 54° 13'$$

and by the Law of Sines,

$$\frac{a}{\sin A} = \frac{b'}{\sin B'}$$

we find the length of side b' to be

$$\frac{5.4}{\sin 22°} = \frac{b'}{\sin 54° 13'}$$

$$b' = \frac{5.4 \sin 54° 13'}{\sin 22°}$$

$$= \frac{5.4(0.81123)}{0.37461}$$

$$= 11.7$$

EXAMPLE: Solve the triangle if it exists, given $C = 125° 48'$, $b = 41.8$, and $c = 56.2$. Give angle accuracy to the nearest minute and side accuracy to two decimal places.

SOLUTION: Since the given angle, C, is obtuse and the side opposite the given angle is larger than the other given side, that is, $c > b$, then one triangle exists. Refer to figure 5-7. By the Law of Sines,

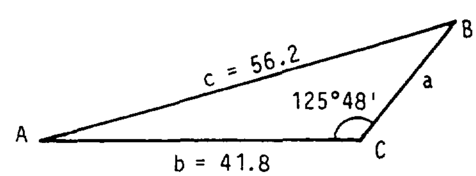

Figure 5-7.—Case 2. Obtuse angle A with $c > b$.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

we get

$$\frac{41.8}{\sin B} = \frac{56.2}{\sin 125° 48'}$$

$$\sin B = \frac{41.8 \sin 125° 48'}{56.2}$$

$$= \frac{41.8(0.81106)}{56.2}$$

$$= 0.60324$$

$$B = 37° 6'$$

Additionally,

$$A = 180° - (B + C)$$

$$= 180° - (37° 6' + 125° 48')$$

$$= 17° 6'$$

and by the Law of Sines,

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

we find the length of side a to be

$$\frac{a}{\sin 17° 6'} = \frac{56.2}{\sin 125° 48'}$$

$$a = \frac{56.2 \sin 17° 6'}{\sin 125° 48'}$$

$$= \frac{56.2(0.29404)}{0.81106}$$

$$= 20.37$$

PRACTICE PROBLEMS:

Use the Law of Sines to solve the remaining parts of triangle ABC given the following parts (give angle accuracy to the nearest minute and side accuracy to one decimal place):

1. $A = 59° 36'$, $B = 48° 14'$, and $c = 86.4$
2. $A = 98° 8'$, $C = 25° 25'$, and $b = 2.1$
3. $B = 30° 30'$, $a = 10$, and $b = 10$
4. $C = 100° 21'$, $a = 4.2$, and $c = 3.2$

ANSWERS:

1. $C = 72° 10'$

   $a = 78.3$

   $b = 67.7$

2. $B = 56° 27'$

   $a = 2.5$

   $c = 1.1$

3. $A = 30° 30'$

   $C = 119°$

   $c = 17.2$

4. No solution; C is obtuse and $c \leq a$.

LAW OF COSINES

Law of Cosines. *In a triangle, the square of any side is equal to the sum of the squares of the other two sides minus twice the* *product of the same two sides multiplied by the cosine of the angle between them*; that is,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

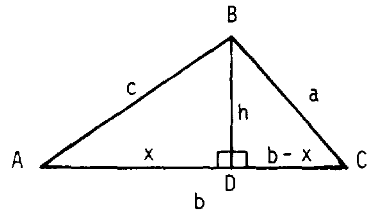

Figure 5-8.—Development of Law of Cosines.

PROOF: Refer to the oblique triangle shown in figure 5-8. Let h be the length of the perpendicular from angle B to the side opposite angle B.

NOTE: $b = b + 0$

$$= b + (x - x)$$

$$= x + (b - x)$$

Considering right triangle ADB formed by h, we obtain

$$\cos A = \frac{x}{c} \text{ or } x = c \cos A$$

and

$$h^2 = c^2 - x^2$$

Substituting the value of x into the last equation gives

$$h^2 = c^2 - c^2 \cos^2 A$$

Considering right triangle CDB, we obtain

$$h^2 = a^2 - (b - x)^2$$

$$= a^2 - b^2 + 2bx - x^2$$

Substituting the value x in the last equation for h^2 gives

$$h^2 = a^2 - b^2 + 2bc \cos A - c^2 \cos^2 A$$

Equating the two values of h^2 gives

$$c^2 - c^2 \cos^2 A = a^2 - b^2 + 2bc \cos A - c^2 \cos^2 A$$

Therefore, rearranging and canceling terms gives

$$a^2 = b^2 + c^2 - 2bc \cos A$$

The same procedure can be applied to derive all three forms of the Law of Cosines, which are

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Case 3. Two Sides and the Included Angle

When two sides and the angle between them are given, we can solve for the remaining parts of the triangle using the Law of Cosines. First, the unknown side is determined; then the two other angles are determined.

EXAMPLE: Solve for the remaining parts of triangle ABC, given $b = 7$, $c = 5$, and $A = 19°$. Give angle accuracy to the nearest minute and side accuracy to one decimal place.

SOLUTION: Refer to figure 5-9. First, find the length of the unknown side using the Law of Cosines,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Hence,

$$\begin{aligned} a^2 &= 7^2 + 5^2 - 2(7)(5) \cos 19° \\ &= 49 + 25 - 70(0.94552) \\ &= 7.8136 \\ a &= \sqrt{7.8136} \\ &= 2.8 \end{aligned}$$

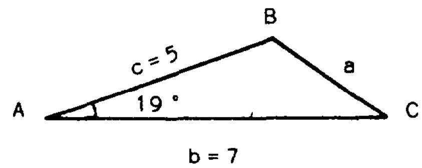

Figure 5-9.—Case 3. Two sides and the included angle.

Next, compute the remaining angles using a rearrangement of the Law of Cosines:

$$\begin{aligned} \cos B &= \frac{a^2 + c^2 - b^2}{2ac} \\ &= \frac{(2.8)^2 + (5)^2 - (7)^2}{2(2.8)(5)} \\ &= \frac{7.84 + 25 - 49}{28} \\ &= -0.57714 \\ &= -\cos 54° 45' \end{aligned}$$

Angle B is an obtuse angle since $\cos B$ is negative. Therefore,

$$B = 125° \, 15'$$

and

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{(2.8)^2 + (7)^2 - (5)^2}{2(2.8)(7)}$$

$$= \frac{7.84 + 49 - 25}{39.2}$$

$$= 0.81224$$

$$C = 35° \, 41'$$

NOTE: Since we are solving angles to the nearest minute, the sum of the angles may not equal exactly 180°.

EXAMPLE: Two points, A and B, are separated by a pond. The distance from A to a third point, C, is 10.2 feet; the distance from C to B is 13.8 feet; and angle C is 52° 40′. Find the distance from A to B to two decimal places and angles A and B to the nearest minute.

SOLUTION: We will first find the distance from point A to point B. Using the Law of Cosines, we find that

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$= (13.8)^2 + (10.2)^2 - 2(13.8)(10.2) \cos 52° \, 40'$$

$$= 190.44 + 104.04 - (281.52)(0.60645)$$

$$= 123.75$$

$$c = \sqrt{123.75}$$

$$= 11.12 \text{ feet}$$

Now we will find angles A and B using the Law of Cosines:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(10.2)^2 + (11.12)^2 - (13.8)^2}{2(10.2)(11.12)}$$

$$= 0.16423$$

$$A = 80° \, 33'$$

and

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{(13.8)^2 + (11.12)^2 - (10.2)^2}{2(13.8)(11.12)}$$

$$= 0.68441$$

$$B = 46^\circ\ 49'$$

Case 4. All Three Sides

The Law of Cosines can also be used to find the size of the angles of a triangle when the length of all three sides are given.

EXAMPLE: Find the measure of each angle (to the nearest minute) of a triangle having sides $a = 7$, $b = 13$, and $c = 14$.

SOLUTION:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{(13)^2 + (14)^2 - (7)^2}{2(13)(14)}$$

$$= 0.86813$$

$$A = 29^\circ\ 45'$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{(7)^2 + (14)^2 - (13)^2}{2(7)(14)}$$

$$= 0.38776$$

$$B = 67^\circ\ 11'$$

and

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{(7)^2 + (13)^2 - (14)^2}{2(7)(13)}$$

$$= 0.12088$$

$$C = 83^\circ\ 3'$$

EXAMPLE: A triangular plot of ground measures 50 meters by 70 meters by 90 meters. Find, to the nearest minute, the size of the angle, A, opposite the longest side.

SOLUTION:

$$\begin{aligned} \cos A &= \frac{(50)^2 + (70)^2 - (90)^2}{2(50)(70)} \\ &= -0.10000 \\ &= -\cos 84^\circ\, 16' \\ A &= 95^\circ\, 44' \end{aligned}$$

PRACTICE PROBLEMS:

Use the Law of Cosines to solve the remaining parts of triangle ABC given the following parts (give angle accuracy to the nearest minute and side accuracy to two decimal places):

1. $a = 54.2$, $c = 83.4$, and $B = 111^\circ\, 11'$
2. $b = 6.6$, $c = 6.6$, and $A = 60^\circ$
3. $a = 22.2$, $b = 33.3$, and $c = 44.4$
4. $a = 15.6$, $b = 16.7$, and $c = 17.8$

ANSWERS:

1. $b = 114.72$
   $A = 26^\circ\, 8'$
   $C = 42^\circ\, 41'$
2. $a = 6.6$
   $B = 60^\circ$
   $C = 60^\circ$
3. $A = 28^\circ\, 57'$
   $B = 46^\circ\, 34'$
   $C = 104^\circ\, 29'$
4. $A = 53^\circ\, 39'$
   $B = 59^\circ\, 34'$
   $C = 66^\circ\, 47'$

AREA FORMULAS

In this section two formulas for finding the area of a triangle will be developed. Recall from plane geometry that the area of a triangle is found by the formula

$$\text{area} = \frac{1}{2}bh$$

where b is any side of the triangle and h is the altitude drawn to that side. While this is a useful formula, it is not a practical one. With the help of trigonometry, we can derive more practical formulas for the area of a triangle.

Consider the triangle in figure 5-8. The length of the altitude is found to be

$$h = c \sin A$$

Substituting this value of h into the geometric area formula results in

$$\text{area} = \frac{1}{2}b(c \sin A)$$

$$= \frac{1}{2}bc \sin A$$

In general, *the area of a triangle is equal to one-half the product of the lengths of any two sides and the sine of their included angle;* that is,

$$\text{area} = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B = \frac{1}{2}bc \sin A$$

EXAMPLE: Find the area of triangle ABC to one decimal place if $a = 13$, $b = 9$, and $C = 40°$.

SOLUTION: Since C is the angle between sides a and b, the area formula is

$$\text{area} = \frac{1}{2}ab \sin C$$

so

$$\text{area} = \frac{1}{2}(13)(9) \sin 40°$$

$$= 58.5(0.64279)$$

$$= 37.6$$

Another formula for the area of a triangle can be derived by the use of the Law of Sines and the previous formula. From the Law of Sines,

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

we find

$$b = \frac{c \sin B}{\sin C}$$

Substituting this value of b into the previous area formula

$$\text{area} = \frac{1}{2}bc \sin A$$

results in

$$\begin{aligned} \text{area} &= \frac{1}{2}\left(\frac{c \sin B}{\sin C}\right)c \sin A \\ &= \frac{c^2 \sin A \sin B}{2 \sin C} \end{aligned}$$

Therefore, *the area of a triangle can be determined if one side and two angles are known* (since the third angle can be found directly); that is,

$$\text{area} = \frac{a^2 \sin B \sin C}{2 \sin A} = \frac{b^2 \sin A \sin C}{2 \sin B} = \frac{c^2 \sin A \sin B}{2 \sin C}$$

EXAMPLE: Find the area of triangle ABC to one decimal place if $A = 25°$, $C = 105°$, and $b = 12$.

SOLUTION: First, we find B to be

$$\begin{aligned} B &= 180° - (A + C) \\ &= 180° - (25° + 105°) \\ &= 50° \end{aligned}$$

The area formula for this situation would be

$$\text{area} = \frac{b^2 \sin A \sin C}{2 \sin B}$$

so,

$$\text{area} = \frac{(12)^2 \sin 25° \sin 105°}{2 \sin 50°}$$

$$= \frac{144(0.42262)(0.96593)}{2(0.76604)}$$

$$= 38.4$$

PRACTICE PROBLEMS:

Find the area of triangle ABC to three decimal places given the following measurements:

1. $b = 20.02$, $c = 40.04$, and $A = 80° 8'$
2. $a = 3.28$, $c = 9.18$, and $B = 42° 21'$
3. $B = 50°$, $C = 70°$, and $c = 5.07$
4. $A = 103° 48'$, $B = 34° 6'$, and $a = 4.24$

ANSWERS:

1. 394.873
2. 10.142
3. 9.074
4. 3.479

SUMMARY

The following are the major topics covered in this chapter:

1. **Oblique triangles:** *Oblique triangles* are triangles containing no right angles. Oblique triangles are made up of either three acute angles or two acute angles and one obtuse angle.

   Acute angles have measures between 0° and 90°.

   Obtuse angles have measures between 90° and 180°.

2. **Law of Sines:** The lengths of the sides of any triangle are proportional to the sines of their opposite angles.

$$\frac{a}{\sin A} = \frac{b}{\sin B} \quad \frac{c}{\sin C}$$

3. **Standard cases for solving oblique triangles using the Law of Sines:**

   Case 1. One side and two angles

   Case 2. Two sides and an angle opposite one of them (This is referred to as the *ambiguous case* since two triangles, one triangle, or no triangle may result from the given data.)

4. **Law of Cosines:** In a triangle, the square of any side is equal to the sum of the squares of the other two sides minus twice the product of the same two sides multiplied by the cosine of the angle between them.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

5. **Standard cases for solving oblique triangles using the Law of Cosines:**

   Case 3. Two sides and the included angle

   Case 4. All three sides

6. **Area of a triangle:**

The area of a triangle is equal to one-half the product of the lengths of any two sides and the sine of their included angle.

$$\text{area} = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B = \frac{1}{2}bc \sin A$$

The area of a triangle can be determined if one side and two angles are known.

$$\text{area} = \frac{a^2 \sin B \sin C}{2 \sin A} = \frac{b^2 \sin A \sin C}{2 \sin B} = \frac{c^2 \sin A \sin B}{2 \sin C}$$

ADDITIONAL PRACTICE PROBLEMS

Use the Law of Sines, Law of Cosines, or area formulas to solve the following problems:

1. To determine the distance from point A to point B across a canyon, Barbara lays off a distance from point C to point B as 440 yards. She then finds that $C = 30° \ 17'$ and $B = 104° \ 53'$. What is the distance, to the nearest yard, between points A and B?

2. Two buoys are 325 feet apart and a boat is 250 feet from one of them. The angle subtended by the two buoys at the boat is $65° \ 10'$. Find the distance, to the nearest foot, from the boat to the other buoy.

3. A triangular tract of land is to be enclosed by a fence. Side a equals 37.25 feet, side c equals 46.98 feet, and the included angle B is $100° \ 30'$. Find the amount of fencing, to the nearest hundredth of a foot, needed to enclose the triangular plot.

4. A 12-foot ladder is placed against an inclined support and reaches 10 feet up the side of the support. The foot of the ladder is 5 feet from the foot of the inclined support. What is the measure of the angle, to the nearest minute, the ladder makes with the support?

5. Find the area, to one decimal place, of a triangular field if two sides of the field are 127 yards and 159 yards and the included angle is $57° \ 18'$.

6. What is the area of a parallelogram, to one decimal place, if the length of one diagonal is 6 inches and the diagonal meets two adjacent sides of the parallelogram at angles with measures $33°$ and $44°$? HINT: Double the area of a triangle.

ANSWERS TO ADDITIONAL PRACTICE PROBLEMS

1. 315 yards
2. 338 feet
3. 149.29 feet
4. 24° 9′
5. 8,496.3 square yards
6. 14.0 square inches